MRI Reconstruction using Data-driven Markov Chain with Joint Uncertainty Estimation

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Synopsis

The application of generative models in MRI reconstruction is shifting researchers' attention from the unrolled reconstruction networks to the probabilistically tractable iterations and permits an unsupervised fashion for medical image reconstruction.

Introduction

The image reconstruction problem is formulated in a Bayesian view and samples are drawn from the posterior distribution given the k-space using the Markov chain Monte Carlo (MCMC) method. The minimum mean square error (MMSE) and maximum a posterior (MAP) estimates are computed. The chains are the reverse of a diffusion process (c.f., Figure 1). Score-based generative models are used to construct chains and are learned from an image database.

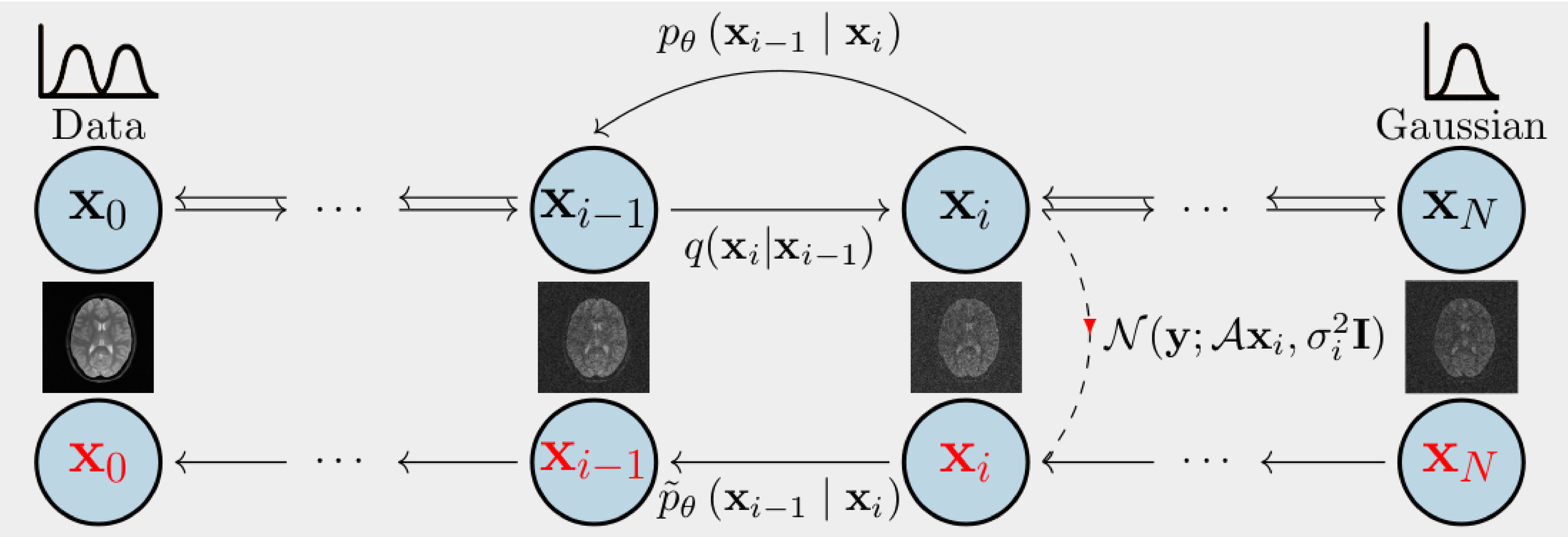

Figure 1. Overview of the proposed method. The unknown data distribution $q(\mathbf{x}_0)$ of the training images goes through repeated Gaussian diffusion and finally reaches a known Gaussian distribution $q(\mathbf{x}_N)$, and this process is reversed by learned transition kernels $p_\theta(\mathbf{x}_{i-1}|\mathbf{x}_i)$. To compute the posterior of the image, a new Markov chain is constructed by incorporating the measurement model into the reverse process (red chain).

Theory

Using Bayes' formula one obtains for each i the desired distribution $p(\mathbf{x}_i \mid \mathbf{y})$ from

$$p(\mathbf{x}_i \mid \mathbf{y}) \propto p(\mathbf{x}_i)\, p(\mathbf{y} \mid \mathbf{x}_i) \quad \text{with} \quad p(\mathbf{y}|\mathbf{x}_i) = \mathcal{CN}(\mathbf{y}; \mathcal{A}\mathbf{x}_i, \sigma_\eta^2 I), \qquad (1)$$

where $\mathcal{A}$ is the parallel MRI forward model, $\mathbf{x}_i$ is the ith image and $\mathbf{y}$ is given k-space data (cf Figure 1). Starting with the initial density $q(\mathbf{x}_N) \sim \mathcal{CN}(0, I)$ at $i = N$, one obtains $p(\mathbf{x}_i)$ with transition kernels $\{p(\mathbf{x}_j|\mathbf{x}_{j+1})\}_{i\leq j<N}$

$$p(\mathbf{x}_i) \propto p(\mathbf{x}_i|\mathbf{x}_{i+1}) \cdot \ldots \cdot p(\mathbf{x}_{N-1}|\mathbf{x}_N) \cdot q(\mathbf{x}_N). \qquad (2)$$

By estimating the transition kernel with the neural network, one obtains the kernel $\{p_\theta(\mathbf{x}_i \mid \mathbf{x}_{i+1})\}_i$ and therefore one can sample $p_\theta(\mathbf{x}_i|y)$ with the unadjusted Langevin Monte Carlo method in order to get an estimate of $\mathbf{x}_i$, i.e.

$$\mathbf{x}_i^{k+1} \leftarrow \mathbf{x}_i^k + \frac{\gamma}{2}\nabla_{\mathbf{x}_i} \log p_\theta(\mathbf{x}_i^k \mid y) + \sqrt{\gamma}\mathbf{z}, \quad z \sim \mathcal{CN}(0, \mathbf{I}), \qquad (3)$$

with stepsize $\gamma > 0$, $k = 1, \ldots, K$ and $\mathbf{x}_i^1 := \mathbf{x}_{i+1}^K$ with $\mathbf{x}_N^1 \sim \mathcal{CN}(0, \mathbf{I})$.

Uncertainty

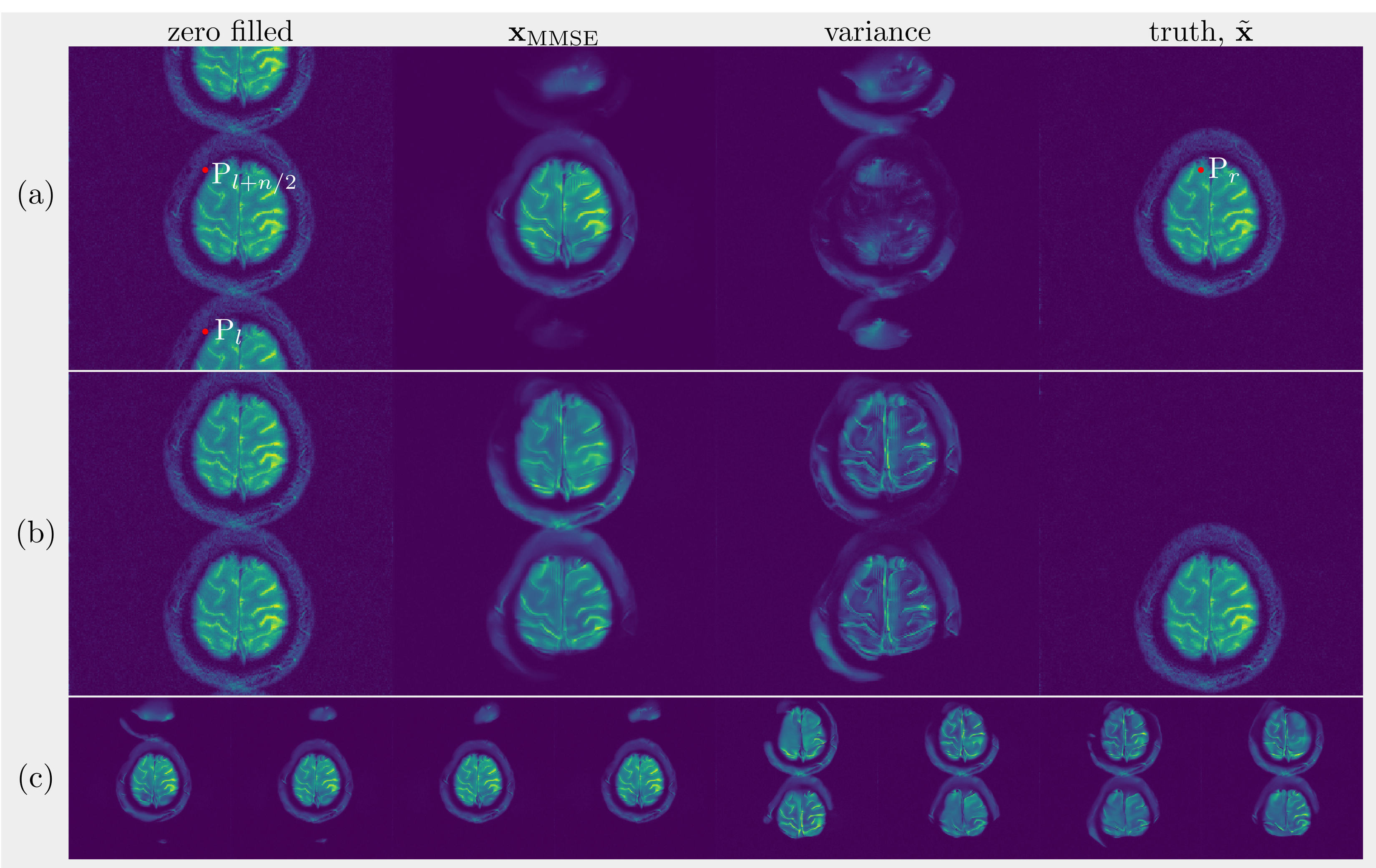

Figure 2. The single-coil k-space is undersampled by skipping every second line. All images in which the superposition of points P_l and $\mathrm{P}_{l+2/n}$ equals to the points P_r in ground truth are solutions to $\mathbf{y} = \mathcal{A}\mathbf{x} + \epsilon$. Aliased images, $\mathbf{x}_{\text{MMSE}}$, variance maps and ground truth are shown. (a) The object is centered. (b) The object is shifted. (c) Selected solutions are presented. The learned reverse process knows less about images that were shifted.

Reference

1. Sohl-Dickstein et al. (2015, ICML) 2. Song et al. (2019, NIPS) 3. Luo et al. (2020, MRM) 4. Song et al. (2021, ICLR) 5. Jalal et al. 2021 (2021, NIPS) 6. Chung et al. 2021 (2022, MIA)

MMSE vs MAP

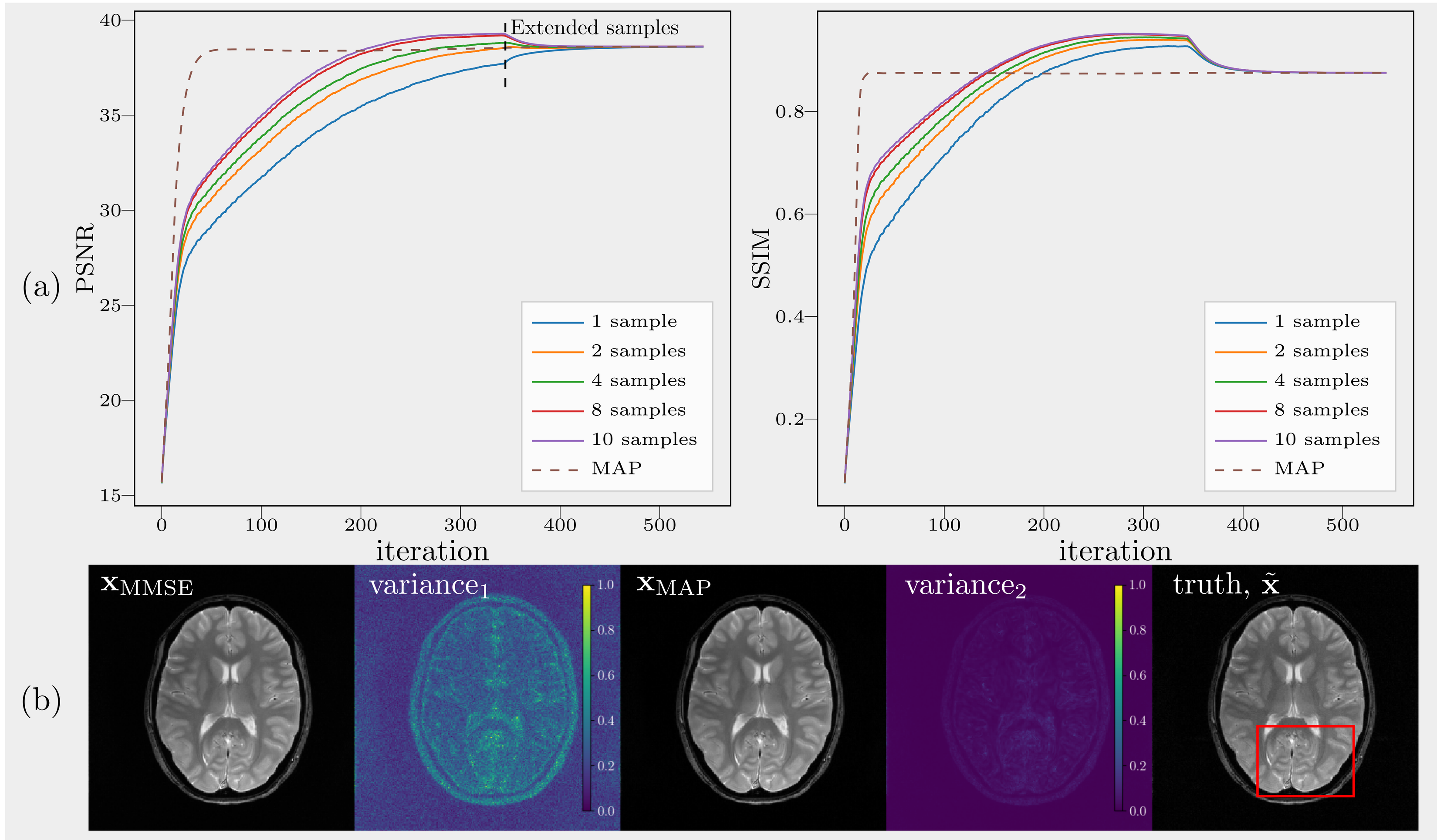

Figure 3. 200 extended iterations after random exploration (i.e, without noise disturbance) and a deterministic estimate of MAP are indicated by solid and dashed lines respectively. (a) PSNR and SSIM over iterations. (b) Variance$_1$ and variance$_2$ were computed from unextended samples and extended samples respectively. Extended samples converge to $\mathbf{x}_{\text{MAP}}$.

Hallucination

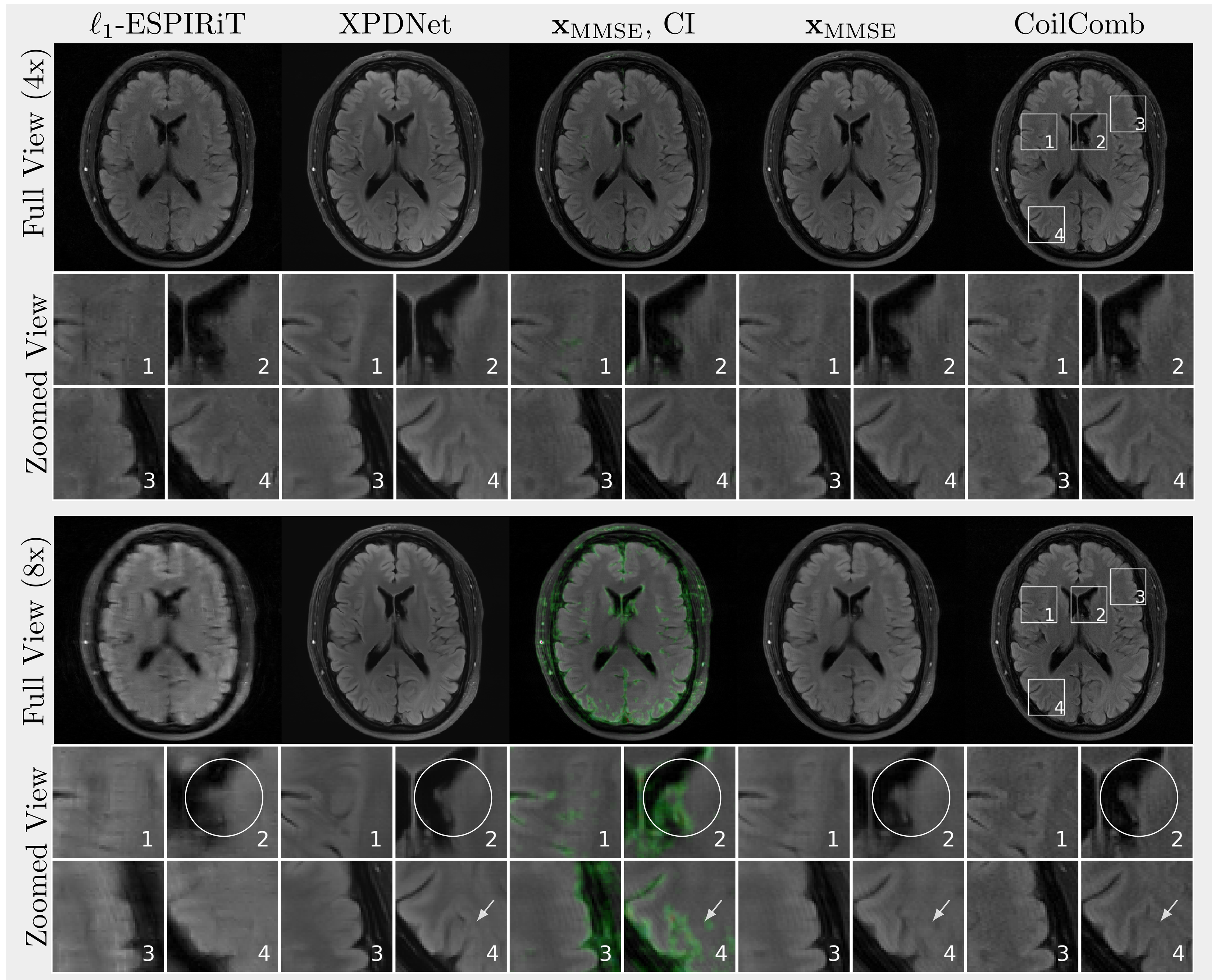

Figure 4. Reconstructions are ℓ_1-ESPIRiT, XPDNet, $\mathbf{x}_{\text{MMSE}}$ highlighted with confidence interval (CI), $\mathbf{x}_{\text{MMSE}}$ and a fully-sampled coil-combined image (CoilComb). Hallucinations appear when using 8-fold acceleration and are highlighted with the CI after thresholding. Selected regions of interests are presented in a zoomed view.

Transferability

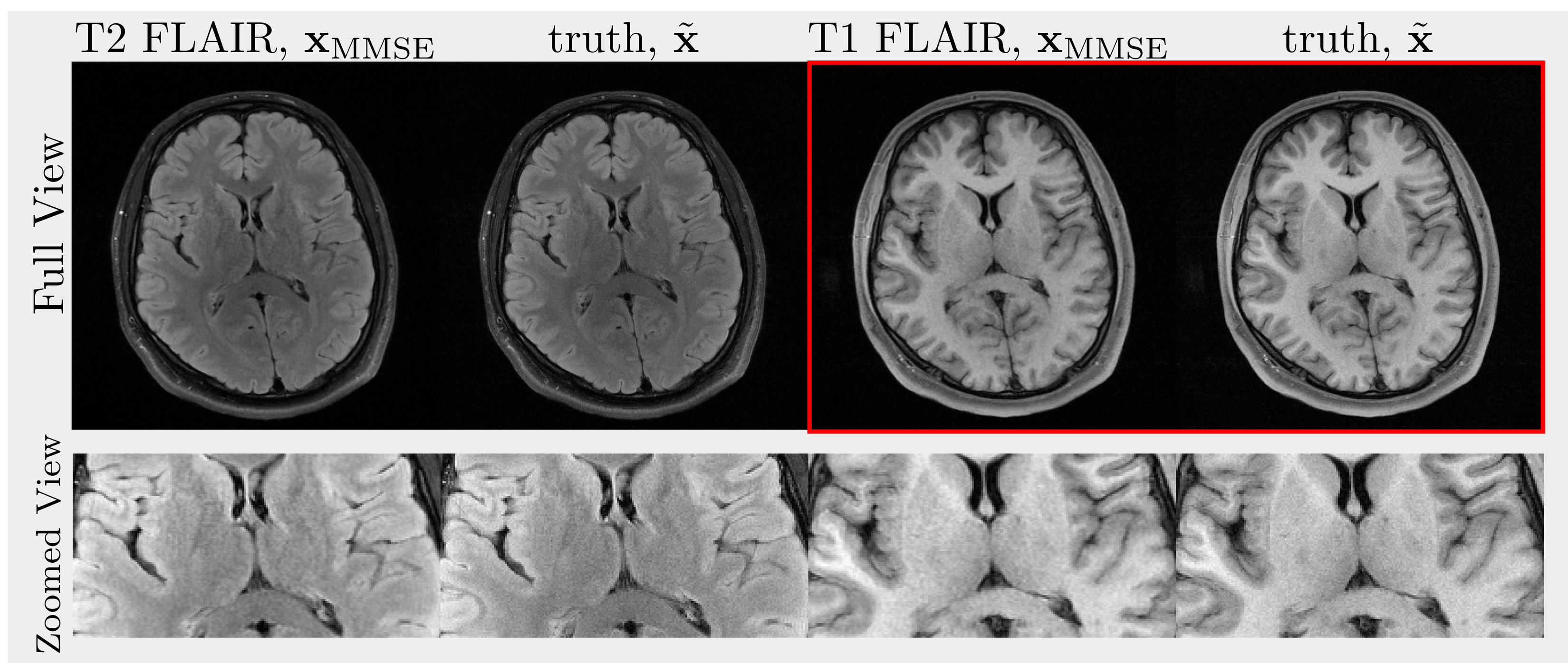

Figure 5. Reconstruction of T2 and T1 FLAIR images (red box) with a network trained on T2 FLAIR images using a Poisson-disk pattern of 8x undersampling in k-space.

Conclusion

This method combines concepts from machine learning, Bayesian inference and image reconstruction. In this setting, the image reconstruction is realized by drawing samples from the posterior term p(x|y) using data driven Markov chains, providing a minimum mean square reconstruction and uncertainty estimation and showing good performance and transferability to different contrasts and sampling patterns.